

Numerical simulation of the stress–strain state of the dental system

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Abstract

We present mathematical models, computational algorithms and software, which can be used for prediction of results of prosthetic treatment. More interest issue is biomechanics of the periodontal complex because any prosthesis is accompanied by a risk of overloading the supporting elements. Such risk can be avoided by the proper load distribution and prediction of stresses that occur during the use of dentures. We developed the mathematical model of the periodontal complex and its software implementation. This model is based on linear elasticity theory and allows to calculate the stress and strain fields in periodontal ligament and jawbone.

The input parameters for the developed model can be divided into two groups. The first group of parameters describes the mechanical properties of periodontal ligament, teeth and jawbone (for example, elasticity of periodontal ligament etc.). The second group characterized the geometric properties of objects: the size of the teeth, their spatial coordinates, the size of periodontal ligament etc. The mechanical properties are the same for almost all, but the input of geometrical data is complicated because of their individual characteristics. In this connection, we develop algorithms and software for processing of images obtained by computed tomography (CT) scanner and for constructing individual digital model of the tooth-periodontal ligament-jawbone system of the patient. Integration of models and algorithms described allows to carry out biomechanical analysis on three-dimensional digital model and to select prosthesis design.

Keywords: Computed tomography, image segmentation, stress-strain state of dental system, numerical simulation

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1. Introduction

Nowadays, theoretical studies of applied problems are performed on the basis of the extensive use of computational tools (computers and numerical methods) [1, 2]. Here, the up-to-date concept of the so-called component-based software [3, 4] is discussed. Component-based software is a set of well-developed software components, which solve individual basic problems. Computational technologies for scientific researches are based on constructing geometrical models, generating computational meshes, applying discretization methods, approximate solving of discrete problems, visualizing, and processing calculated data.

Information technology are used in all areas of medicine, and dentistry is not exception [5]. Unfortunately, often most doctors use computers only to maintain office and medical documentation, and take it as the information technology. At the same time, information technologies are widely used in dentistry for diagnostics: digital X-rays, digital photos, scanned diagnostic model. The production of dental prostheses based on computer milling [6] as well as various system in the gnathology are developing very rapidly.

In our view, it is very promising the development of software that combine processing various images of dental system and subsequent treatment planning. To justify the choice of designs of dental prostheses and orthodontic appliances the methods of mathematical modelling can be used. The essence of this methods is the ability to predict and evaluate the effect of medical intervention using the calculation of stress-strain state of prostheses and appliances as well as tissues and organs of dental system. Unfortunately, the use of mathematical modelling methods is usually limited to research with practical recommendations. In our view, this approach has right to life. However, the rapid development of information technology and the various methods of diagnosis allows us to bring the mathematical modelling to the doctor.

The use of mathematical modelling in practical work of dentists is limited to a number of reasons, the main of which is the lack of appropriate software. Most of the mathematical models described in the literature are implemented using applied software packages for finite element analysis. Operation with them requires special skills and knowledge as well as involvement of specialists in mathematics. Moreover, these software packages are aimed at solving a wide range of problems, which greatly increases their price.

In this paper we present the software package that allows to carry out the biomechanical analysis of dental system. The basis of the software is the mathematical model that allows to calculate the stresses and displacements in teeth, periodontal ligament and jawbone for the given geometric models and loads on the prosthesis. The software provides an opportunity of constructing the geometrical model of the dental system based on processing CT images.

2. Applied software

Traditionally we recognize two types of software: applied software and system software. The latter is supporting software for developing general-purpose applications, which is not directly related to applied problems. Below we consider problems associated with software for numerical analysis of applied mathematical models. We discuss both commercial and free/open source software for multiphysical simulations.

2.1. Features of applied software

At early stages of using computers, mathematical models were enough simple and insufficiently reliable. Applied software, in fact, were primitive, too. The transition to a new problem or new version of calculations required assembling practically a new program, which is an essential modification of the previously developed codes.

The modern state of using computational tools is characterized by studying complex mathematical models. Therefore, software expands greatly, becomes large as well as difficult in study and use. Contemporary software includes a large set of various program units, which require to construct a certain workflow for efficient employment.

In numerical simulations, we do not study a particular mathematical problem, but we investigate a class of problems, i.e., we highlight some order of problems and hierarchy of models. Therefore, software should be focused on multitasking approach and be able to solve a class of problems via quick switching from one problem to another.

Numerical analysis of applied problems is based on multiparametric predictions. In the framework of some specific mathematical model, it is necessary to trace the impact of various parameters on the solution. This feature of numerical simulations requires that software have been adapted to massive calculations.

Thus, software developed for computational experiments must, on the one hand, be adapted to quick significant modifications, and, on the other hand, be sufficiently conservative to focus on massive calculations using a single program.

Our experience shows that applied software is largely duplicated during its lifecycle. To repeat programming a code for a similar problem, we waste time and eventually money. This problem is resolved by the unification of applied software and its standardization both globally (across the global scientific community) and locally (within a single research team).

2.2. Modular structure

A modular structure of applied software is primarily based on a modular analysis of the area of applications. Within a class of problems, we extract relatively independent sub-problems, which form the basis to cover this class of problems, i.e., each problem of the class can generally be treated as a certain construction designed using individual sub-problems.

An entire program is conceptually represented as a set of modules appropriately connected with each other. These modules are relatively independent

and can be developed (coded and verified) separately. The decomposition of the program into a series of program modules implements the idea of structured programming.

A modular structure of programs may be treated as an information graph, which vertices are identified with program modules, and branches (edges) correspond to the interface between modules.

The functional independence and content-richness are the main requirements to program modules. A software module can be associated with an application area. For example, in a computational program, a separate module can solve some meaningful sub-problem. Such a module can be named a subject-oriented module.

A software module can be connected with the implementation of some computational method, and therefore the module can be called a mathematical (algorithmic) module (solver). In fact, it means that we conduct a modular analysis (decomposition) at the level of applications or computational algorithms.

Moreover, mathematical modeling is based on studying a class of mathematical models and, in this sense, there is no need to extract a large number of subject-oriented modules. A modular analysis of a class of applied problems is carried out with the purpose to identify functionally independent individual mathematical modules.

Software modules may be parts of a program, which are not directly related to some meaningful in applied or mathematical sense sub-problem. They can perform some supporting operations and functions. This type of modules, referred to as internal ones, includes data modules, documentation modules, etc.

Separate parts of the program are extracted for the purpose of autonomous (by different developers) designing, debugging, compiling, storage, etc. A module must be independent (relatively) and easily replaced. When we create an applied software for computational experiments, we can highlight such a characteristics of software module as actuality, i.e., its use as much as possible in a wider range of problems of this class.

2.3. Basic components

The structure of applied software is defined according to solving problems. For modeling multiphysics processes, both general-purpose and specialized software packages are used.

The basic components of software tools for mathematical modeling are the following:

- pre-processor — preparation and visualization of input data (geometry, material properties), assembly of computational modules,
- processor — generation of computational grids, numerical solving discrete problems,
- post-processor — data processing, visualization of results, preparation of reports.

Below we present an analysis of these components.

2.4. Data preparing

In research applied software, the data input is usually performed manually via editing text input files. A more promising technology is implemented in commercial programs for mathematical modeling. It is associated with the use of a graphical user interface (GUI).

To solve geometrically complex problems, it is necessary to carry out an additional control for input data. According to modular structure of applied software, the problem of controlling geometric data is resolved on the basis of unified visualization tools.

The core of a pre-processor is a task manager, which provides assembling an executable program for a particular problem. It is designed for automatic preparation of numerical schemes for solving specific problems. The task manager includes system tools for solving both steady and time-dependent problems as well as adjoint problems using a multiple block of computational modules. A numerical scheme for adjoint problems includes an appropriately arranged chain of separate computational modules.

2.5. Computational modules

Software packages for applied mathematical modeling consist of a set of computational modules, which are designed to solve specific applied problems, i.e., to carry out the processor functionality (generating mesh, solving systems of equations). These software modules are developed by different research groups with their own traditions and programming techniques.

A program package for applied mathematical modeling is a tool for integrating the developed software in a given application area. The possibility of integrating a computational module into a software system for mathematical modeling, which makes possible automated assembling computational schemes from separate computational modules, is provided by using unified standards of input/output.

A computational module is designed to solve a particular applied problem. For transient problems, a computational module involves solving the problem from an initial condition up to the final state. The reusability of computational modules in different problems results from the fact that the computational module provides a parametric study of an individual problem. To solve the problem, we choose a group of parameters (geometry, material properties, computational parameters, etc.), which can be varied.

System tools of the software system provide a user-friendly interface to computational modules on the basis of dialog tools for setting parameters of a problem.

2.6. Data processing and visualization

A post-processor of a software system serves for visualization of computational data. This problem is resolved by using a unified standard for the output of computational modules.

Software systems for engineering and scientific calculations require visualization of 1D, 2D, and 3D calculated data for scalar and vector fields.

Data processing (e.g., evaluation of integral field values or critical parameters) is performed in separate computational modules. Software systems for mathematical modeling make possible to calculate these additional data and to output the results of calculations as well as to include them in other documents and reports.

2.7. User-friendly interface

In modern applied software, the user interaction with a computer is based on a graphical user interface (GUI) is a system of tools for user interaction with a computer, which is based on the representation of all available user system objects and functions in the form of graphic display components (windows, icons, menus, buttons, lists, etc.). The user has the random access (using keyboard or mouse) to all visible display objects.

GUI is employed in commercial software for mathematical modeling, such as ANSYS¹, Marc, SimXpert and other products of MSC Software², STAR-CCM+ and STAR-CD from CD-adapco³, COMSOL Multiphysics⁴. In this case, preparing geometrical models, selecting equations, setting boundary and initial conditions is conducted in the most easy and clear way for inexperienced users.

2.8. Component-based implementation of functionalities

Functionality of modern software systems for applied mathematical modeling must reflect the achieved level of the development of the theory and practice for numerical algorithms and software. This goal is achieved by component-oriented programming, which is based on using well developed and verified software for solving basic mathematical problems (general mathematical functionality).

In applied software packages, the actual content of computational problems is the solution of initial value problem (the Cauchy problem) for systems of ordinary differential equations, which reflects mass, momentum, and energy conservation laws. We come to Cauchy problems by discretization in space (using finite element methods, control volume or finite difference schemes). The general features of the ODE system are the following:

- it is nonlinear,
- it is coupled (some variables are dependent on other ones),
- it is stiff, i.e., different scales (in time) are typical for the considering physical processes.

¹<http://www.ansys.com>

²<http://www.mscsoftware.com>

³<http://www.cd-adapco.com>

⁴<http://www.comsol.com>

To solve the Cauchy problem, we need to apply partially implicit schemes (to overcome stiffness) with iterative implementation (to avoid nonlinearity) as well as partial elimination of variables (to decouple unknowns).

Applied software allows to tune iterative processes in order to take into account specific features of problems: to arrange nested iterative procedures, to define specific stopping criteria and time step control procedures and so on. In this case, computational procedures demonstrate a very special character and work in a specific (and often very narrow) class of problems; the transition to other seemingly similar problems can decrease its efficiency drastically.

2.9. Computational components

Obviously, it is necessary to apply more efficient computational algorithms in order to implement complicated nonlinear multidimensional transient applied models taking into account parallel architecture (cluster, multicore etc.) of modern computing systems. The nature of used algorithms must be purely mathematical, i.e., without involving any other considerations.

Here is the hierarchy of numerical algorithms complexity increasing top down:

- linear solvers – direct methods and, first of all, iterative methods if systems of equations have large dimensions,
- nonlinear solvers – general methods for solving nonlinear systems of algebraic equations,
- ODE solvers – for solving stiff systems of ODEs appearing in some mathematical models.

Solvers must support both sequential and parallel implementations and also reflect modern achievements of numerical analysis and programming techniques. It means, in particular, that we need to use appropriate specialized software developed by specialists in numerical analysis. This software must be deeply verified in practices and greatly appreciated by international scientific community.

Such applied software systems are presented, in particular, in the following collections:

- Trilinos⁵ – Sandia National Laboratory,
- SUNDIALS⁶ – Lawrence Livermore National Laboratory,
- PETSc⁷ – Argonne National Laboratory.

⁵<http://trilinos.sandia.gov>

⁶<https://computation.llnl.gov/casc/sundials/main.html>

⁷<http://www.mcs.anl.gov/petsc>

3. Construction of digital geometric model

To carry out the mathematical modelling stress-strain state of dental system we have to construct digital geometric model containing all the computational domains.

3.1. Segmentation of CT images

The input data for construction of the digital geometric model is CT images. Using this data we need to obtain the computational domain containing sub-domains described below (Section 4). Processing CT images of dental system and constructing digital geometric models of teeth and jawbone is quite challenging. This is due to similar X-ray properties of teeth and jawbone, as well as low (for identification of periodontal ligament) resolution of contemporary CT scanners. The process of selecting objects in image is called *segmentation*. We perform segmentation as follows:

1. Selection of the region of interest.
2. Separation of entire dentition (as a single object *Dentition*) and upper or lower jawbone (as object *Jaw*)
3. Partitioning the object *Dentition* into series of individual objects *Tooth_{XX}*.

In the first step dentist selects a region of interest on the loaded CT image.

The second step is constructing models of the dentition and jawbone using threshold transform and watershed transform with markers. Let us give a brief description of the used segmentation algorithms.

3.2. Computational segmentation algorithm

Threshold transform is the basis in applied problems of segmentation. Suppose that histogram shown in Figure 1 corresponds to some image $f(x, y, z)$ containing bright objects on a dark background, so that the brightness values of the object and the background pixels are concentrated near some prevailing value. The obvious way to select an object from the surrounding background is to choose the threshold level delineating the distribution of brightness values. In our software user can set the threshold value T (as shown in Figure 1).

In this case, the result of the threshold transform will contain only the most optically dense part of pixels in the image of dental system: teeth and dense jawbone (Figure 2, the area colored pink).

Further, to select dentition from jawbone we use approach based on so-called *morphological watershed transform with markers* that often give more stable results of the image segmentation. Here we give only description of the approach. Justification and detailed exposition of the algorithm can be found in [7, 8].

The concept of watershed is based on representation of the image as three-dimensional surface, given by two spatial variables and brightness level as height of the surface (terrain). In such a “topographic” interpretation we consider the points of three types: (a) local minimum points; (b) point on the slopes,

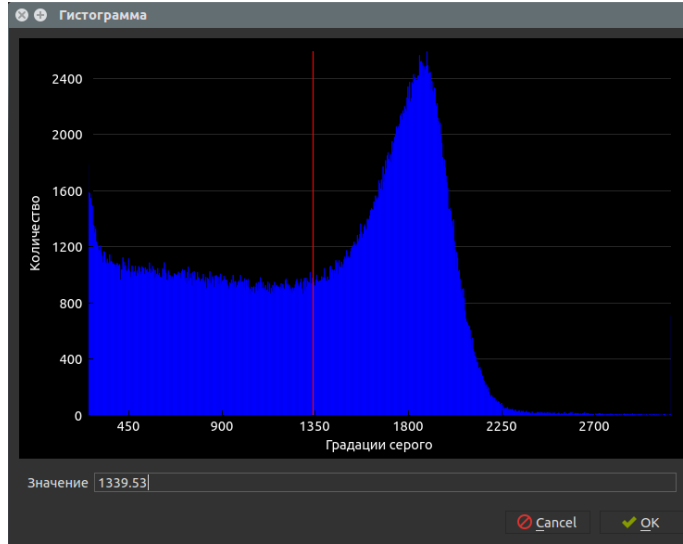


Figure 1: Image histogram

from which the water slides in the same local minimum; and (c) points on the ridge, from which the water slides in more that one local minimum with equal probability. With regard to specific local minimum the set of points satisfying condition (b) is called *catchment basin* of this minimum. The set of points satisfying condition (c) form ridge lines on the surface of terrain and are called *watershed lines*.

The main purpose of the segmentation algorithms based on concepts introduced is to find watershed lines. The basic idea of the method seems simple. Suppose that hole is pierced in every local minimum. Then each local minimum becomes the source of a lake. The entire terrain is progressively flooded through holes and lakes eventually meet neighbouring lakes. Virtual dams are constructed to keep the neighboring lakes as the water level rises. When the image surface is completely flooded the virtual dams correspond to the watershed lines.

Direct application of the watershed transform usually leads to excessive segmentation caused by noise and other local irregularities in the image. Wherein excessive segmentation becomes so significant that makes the result practically useless. In our case, this means a large number of domains identified during segmentation. The practical solution of this problem is to limit allowable number of domains by including in the procedure preprocessing step, which serves to introducing additional knowledge.

Approach used to control excessive segmentation is based on the use of *markers*. Marker is a connected component belonging the image. We distinguish internal markers, belonging to the object of interest, and external markers, related to the background. In our case, the input image is the image obtained

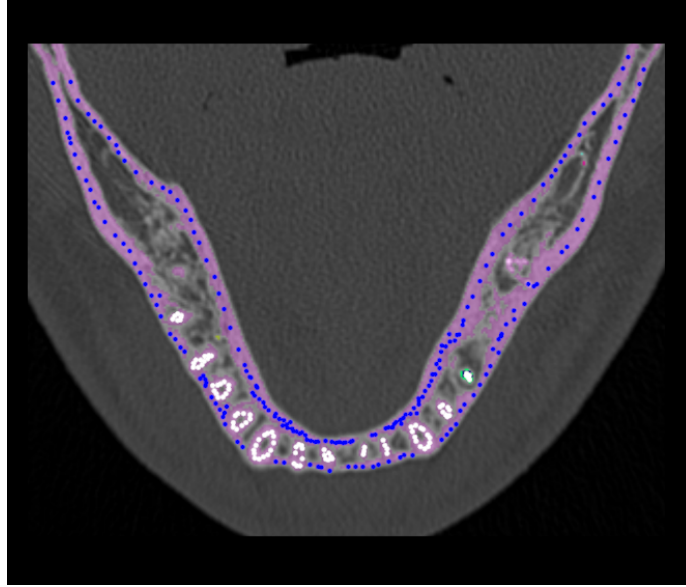


Figure 2: Result of the threshold transform and setting markers

by threshold transform and containing jawbone and dentition only. Therefore, the object *Dentition* corresponds to the internal markers and the object *Jaw* matches the external markers. At that, setting of markers are carried out by user. Figure 2 illustrates set of markers on one slice of CT image.

3.3. Software implementation

For software implementation of the segmentation based on watershed transform with markers we use cross-platform open source software **Insight Segmentation and Registration Toolkit (ITK)** ⁸ that provides users and developers by the tools of image processing [9].

The resulting rough model of dentition (object *Dentition*), which is obtained by watershed transform, requires clarification. Since the teeth are normally in contact on the proximal surfaces, then the object *Dentition* is a single object without division into individual teeth. Therefore the next step is cutting the object *Dentition*. This step is implemented as a special tool, by which the user cuts the object *Dentition* and assigns a label to each tooth.

As a result of segmentation it is obtained the image containing object *Jaw* and series of objects *Tooth_XX* (*XX* is corresponding number of the tooth). This image is saved in NIfTI format ⁹, which is developed by NIfTI Data Format Working Group and being the adaptation of the well-known Analyze 7.5 format.

⁸<http://www.itk.org>

⁹<http://niftilib.sourceforge.net>

4. Mathematical model of the stress–strain state of dental system

The problem of finding the strain field in dental system is static elasticity problem. Governing equations of the elasticity are the momentum balance equation and constitutive equations of elastic medium [10, 11].

4.1. Governing equation

In the case of small strains the momentum balance equation has the following form:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \operatorname{div} \boldsymbol{\sigma}, \quad (1)$$

where t is time, \mathbf{u} is displacement vector, ρ is density, $\boldsymbol{\sigma}$ is stress tensor.

The left-hand side of equation 1 (which is inertial term) becomes significant only in the case of acoustic phenomena, i.e. at times specific to the period of acoustic properties of the oscillations. Therefore, in the case of slowly varying loads, inertial term can be omitted and equation (1) takes the form

$$\operatorname{div} \boldsymbol{\sigma} = 0. \quad (2)$$

The constitutive equations of elastic medium define the relation between stress and strain ($\boldsymbol{\varepsilon}$) tensors as follows

$$\boldsymbol{\varepsilon} = \frac{1}{2} \left(\operatorname{grad} \mathbf{u} + (\operatorname{grad} \mathbf{u})^T \right), \quad (3)$$

where \mathbf{u} is the strain vector field determined for all points of the medium, subscript T denotes the transpose of the tensor.

Hooke's law in three-dimensional case can be written as follows

$$\boldsymbol{\sigma} = 2\mu(\mathbf{x})\boldsymbol{\varepsilon} + \lambda(\mathbf{x}) \operatorname{div} \mathbf{u} \mathbf{I}, \quad (4)$$

where $\lambda(\mathbf{x})$ and $\mu(\mathbf{x})$ are the Lamè coefficients, \mathbf{I} is the second-rank identity tensor.

Modeled object consists of jawbone (Ω_1), periodontal ligament (Ω_2), teeth, which is supports for bridge prosthesis (Ω_3), and bridge prosthesis (Ω_4). Therefore Lamè coefficients are piecewise constant:

$$\lambda(\mathbf{x}) = \lambda_\alpha, \quad \mathbf{x} \in \Omega_\alpha, \quad (5)$$

$$\mu(\mathbf{x}) = \mu_\alpha, \quad \mathbf{x} \in \Omega_\alpha. \quad (6)$$

Equations (2)–(4) are the equations of the elasticity theory, which is supplemented by the boundary conditions on the outer surface of the modeled object.

4.2. Boundary conditions

To compute stresses arising in dental system under loads on prostheses we do not need consider whole jawbone, it is known that elastic stresses, arising from the local loads, decay at distance about the size of the area of load application. Thereby we may consider only part of the jawbone extending for distance about 1.5–2 length of the tooth root from the prosthesis supporting teeth. Therefore we cut the jawbone by some planes. Thus we get the jaw fragment, the size of which is determined according to the above criteria.

Then outer surface Γ of the jaw fragment separate into three regions with different external influences:

1. free surface where external stresses are absent:

$$(\boldsymbol{\sigma} \cdot \mathbf{n})(\mathbf{x}) = 0, \quad \mathbf{x} \in \Gamma_1; \quad (7)$$

2. surface where strains are absent:

$$\mathbf{u}(\mathbf{x}) = 0, \quad \mathbf{x} \in \Gamma_2; \quad (8)$$

3. surface where distributed external load are given:

$$(\boldsymbol{\sigma} \cdot \mathbf{n})(\mathbf{x}) = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} \in \Gamma_3. \quad (9)$$

4.3. Finite element discretization

Let us perform discretization of problem (2)–(9) using finite element method. First, we need to get a variational form of problem (2)–(9). Let us introduce standard Hilbert space of scalar functions $L_2(\Omega)$ with the following inner product and norm:

$$(u, v) = \int_{\Omega} u(\mathbf{x})v(\mathbf{x})d\mathbf{x}, \quad \|u\| = (u, u)^{1/2}.$$

For vector functions we define $\mathbf{L}_2(\Omega) = [L_2(\Omega)]^3$. In addition, let $H^1(\Omega)$ and $\mathbf{H}^1(\Omega)$ be Sobolev spaces of scalar and vector functions, respectively.

Further, we define the spaces of trial and test functions as follows:

$$\mathbf{V} = \{\mathbf{v} \in \mathbf{H}^1(\Omega) : \mathbf{v}(\mathbf{x}) = 0, \mathbf{x} \in \Gamma_2\}.$$

Multiplying both sides of (2) by $\mathbf{v} \in \mathbf{V}$ and integrating the result by part, we get:

$$\int_{\Omega} \boldsymbol{\sigma}(\mathbf{u})\boldsymbol{\varepsilon}(\mathbf{v})d\mathbf{x} = \int_{\Gamma_3} (\mathbf{f}, \mathbf{v})d\mathbf{s}, \quad (10)$$

where $\mathbf{u} \in \mathbf{V}$.

For numerical solving we need to transfer continuous variational problem(10) to discrete one. Let tetrahedron mesh Ω_h is generated. Further on the mesh Ω_h we introduce [12] finite-dimensional subspaces of trial and test functions:

$\mathbf{V}_h \subset \mathbf{V}$ and define the following discrete variational problem: find $\mathbf{u}_h \in \mathbf{V}_h$ such that

$$\int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}_h) \boldsymbol{\varepsilon}(\mathbf{v}_h) d\mathbf{x} = \int_{\Gamma_3} (\mathbf{f}, \mathbf{v}_h) d\mathbf{s}, \quad \forall \mathbf{v}_h \in \mathbf{V}_h. \quad (11)$$

Solver of the obtained discrete problem (11) is implemented using library DOLFIN [13] of the software package FEniCS¹⁰.

5. Software description

We develop software **Computer Dental System** (CDS) designed for use by prosthodontists at various levels of dental treatment.

5.1. Main elements

The presented software package allows to carry out individual biomechanical analysis of dental system of patient on the basis of CT images. Originality of the software lies in the fact that it combines the following functionality: medical records, imaging with construction of the digital models and numerical simulation of stress-strain state of dental system based on which treatment (prosthetics) option is chosen.

Graphical user interface is implemented using Qt¹¹, crossplatform framework for development software in C++ [14, 15]. In addition for visualization and processing 3D objects we use Visualization Toolkit (VTK)¹², crossplatform applied software for 3D modelling and visualization [16].

The main window of the presented software contains the following tabs:

- *Patients*
- *Segmentation*
- *Prostheses*
- *Calculation*

First, a doctor performs inspection of patient: carries out external examination, determines the state of the temporomandibular joint, estimates the occlusion, the oral mucosa and the state of periodontal ligament and hard tissue of teeth. The results of the examination are recorded in the patient's history, which is filled on the tab *Patients* (see Figure 3)

According to the results of examination doctor makes the diagnosis and preliminary treatment plan. At this stage, it is necessary to solve the questions concerning the conduct of professional oral hygiene, extraction of teeth that cannot be treated and restored. In addition, if necessary, actions on surgical

¹⁰<http://fenicsproject.org>

¹¹<http://qt.io>

¹²<http://www.vtk.org>

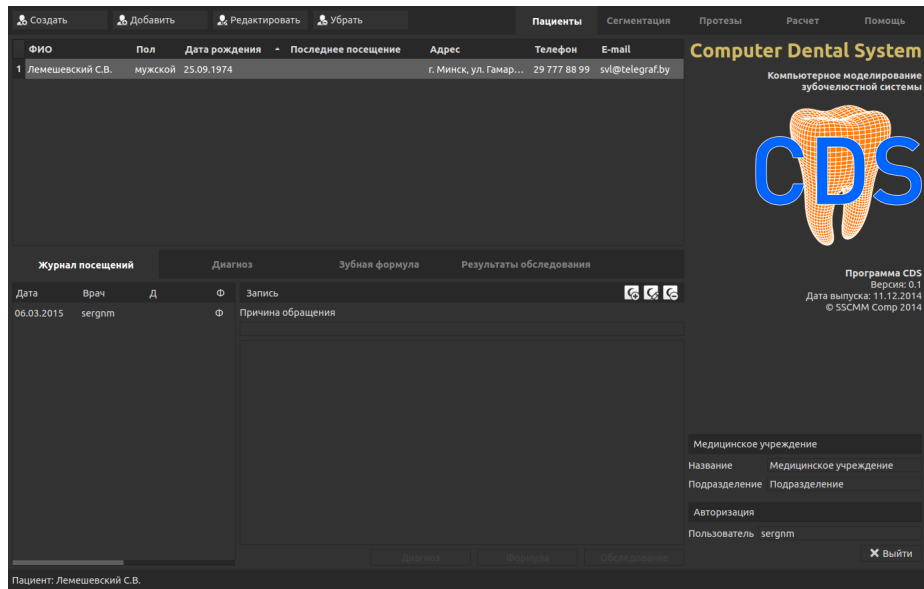


Figure 3: Tab *Patients*

preparation of the oral cavity for prosthetics are determined. Plan of endodontic treatment of the supporting teeth is drawn up only after final selection of the design of prosthesis. At this stage doctor also preselect several future designs of the prostheses.

5.2. Computational kernel

Biomechanical analysis and further choice of the design of prosthesis is performed using 3D digital model of the dental system of patient. Digital model constructed as described above includes all teeth and jawbone. Figure 4 illustrates the tab *Segmentation*, where doctor can construct the digital geometric model using CT images.

Because of insufficient resolution of contemporary CT scanners periodontal ligament is not contrasted in the image. Therefore, domain of periodontal ligament constructed when mesh is generated after doctor specify the possible prostheses.

When the geometric models of jawbone and teeth are constructed, doctor proceeds to construction of prostheses that have been identified at the stage of examination and diagnosis. This feature is implemented on the tab *Prostheses* shown in Figure 5. Here doctor can add prostheses selecting supporting teeth. Moreover, for each tooth doctor can set the degree of mobility and thickness of periodontal ligament. Each degree of mobility corresponds to its own Lamé coefficients in the domain of periodontal ligament adjacent to corresponding tooth.

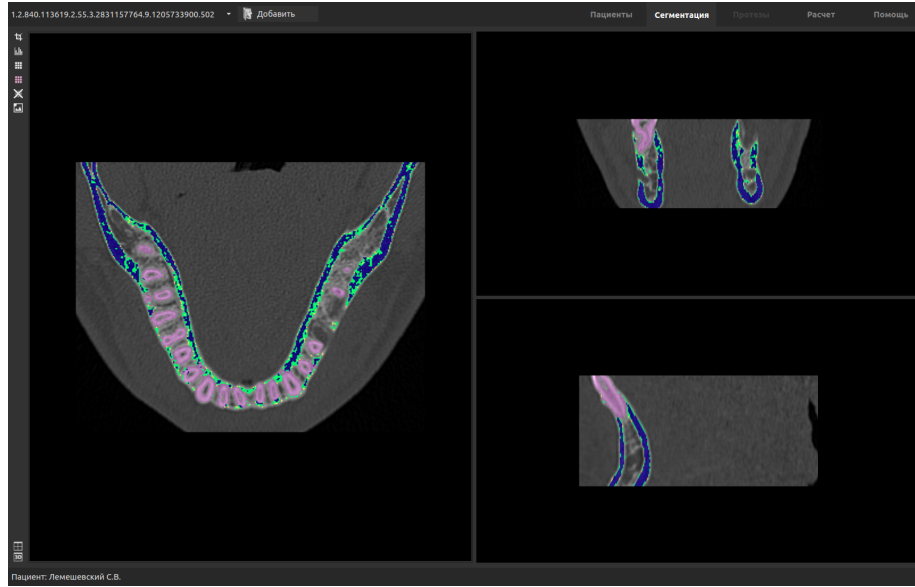


Figure 4: Tab *Segmentation*

When possible prostheses are constructed doctor can go to performing calculation and analysis of the results. This actions are performed on the tab *Calculation* (Figure 6).

When we select tab *Calculation*, it is automatically generated using constructed prostheses the modelling domains: only part of jawbone and supporting teeth are cut out and prostheses are added.

Here doctor generate computational meshes for all options of prosthetics. In addition, domain of periodontal ligament with the given thickness is also generated. Moreover, the standard surfaces of loads are automatically marked: outer supporting teeth and center of prosthesis (see Figure 7). The standard load is equal to 100 MPa and normal to the surface. Such loads are equivalent to force of 10 kg required for mastication.

On the tab *Calculation* the results are visualized for performing biomechanical analysis. After biomechanical analysis of various designs of prostheses and comparing the obtained stress-strain states, doctor choose the design of prosthesis with the less negative impact on dental system. For biomechanical analysis the fields of strains and stresses are visualized. In addition, it is implemented displaying slices along planes. Moreover, the maximum near the teeth of the computed fields are shown.

Analysis of the proposed design of prosthesis is carried out on the basis of investigation of the strains and stresses (see Figures 8 and 9).

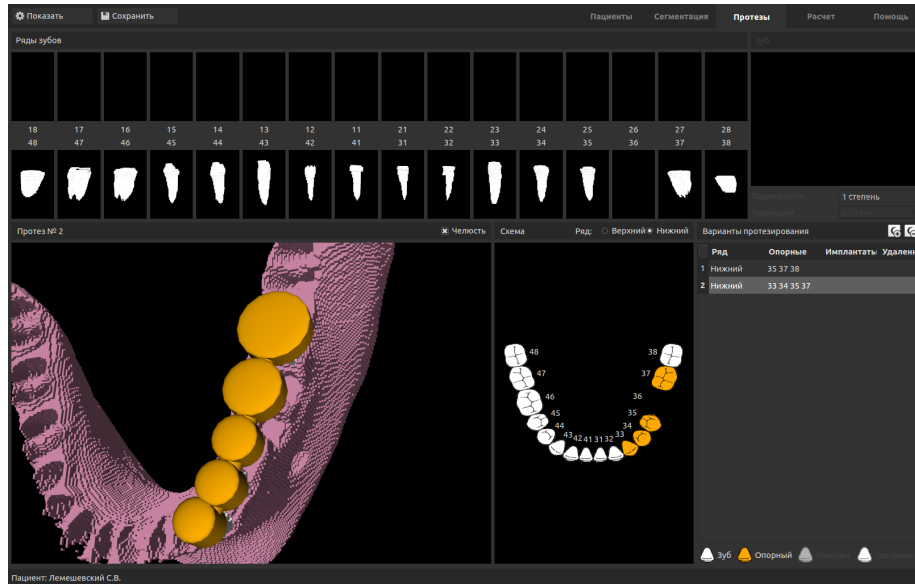


Figure 5: Tab *Prostheses*

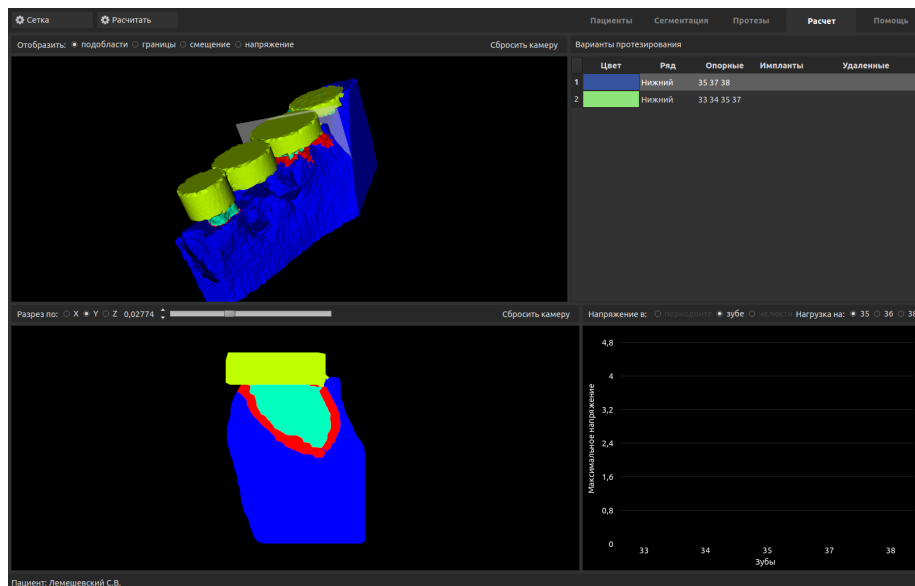


Figure 6: Tab *Calculation*

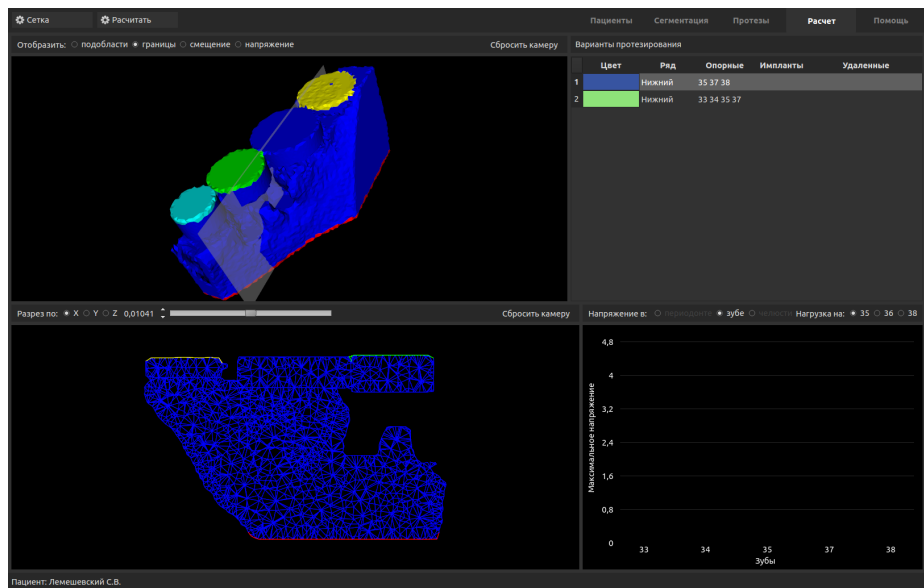


Figure 7: Boundary surfaces where loads are given

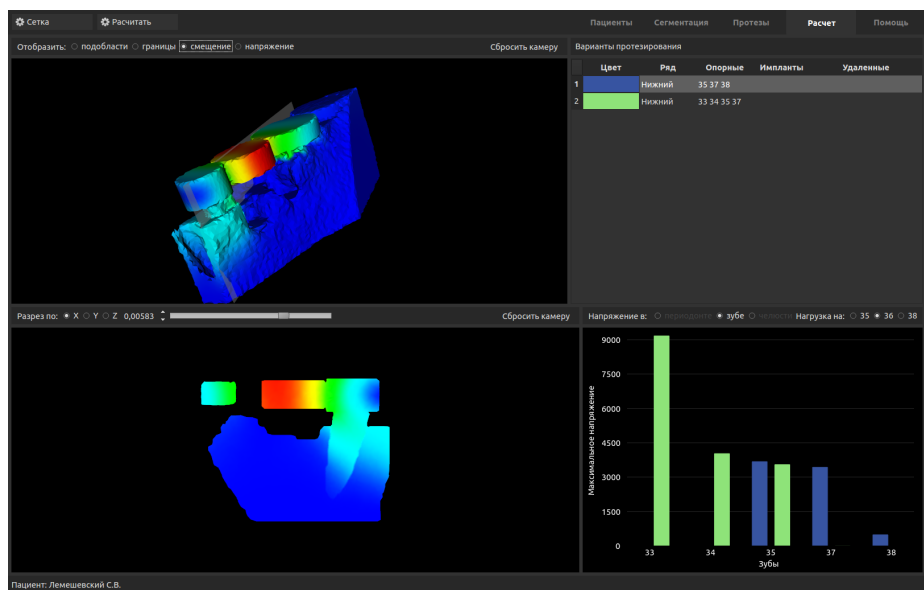


Figure 8: Calculation results: strain field

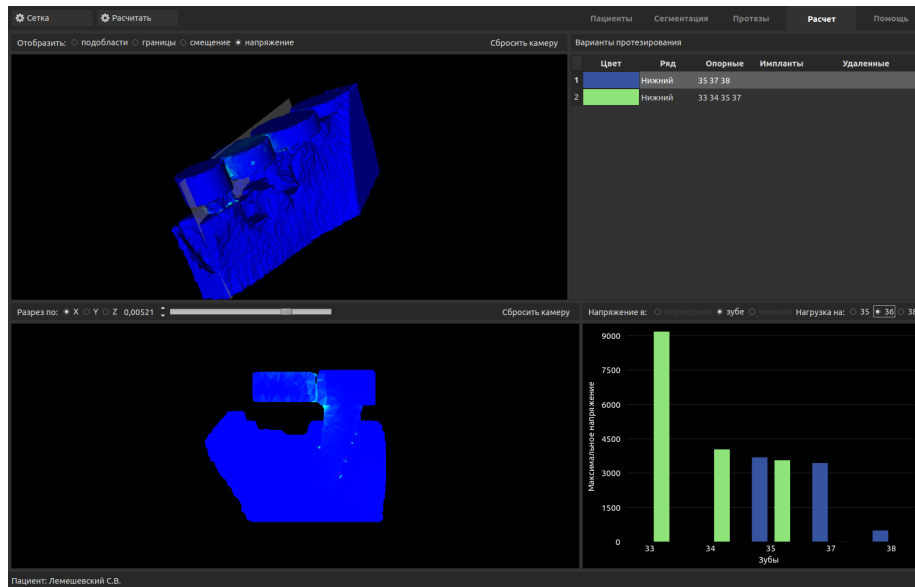


Figure 9: Calculation results: stress field

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